INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 41, Northern Spring 2020 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Squareland is a 6×6 grid, where each 1×1 cell is either a kingdom or a disputed territory. There are 27 kingdoms and 9 disputed territories in Squareland. Claimants to each disputed territory are those kingdoms that share an edge or a vertex with that territory. Is it possible that for every pair of disputed territories the numbers of claimants to each of them are different? (4 points)

2. What is the maximum number of distinct integers that can be written in a row such that the sum of any 11 consecutive numbers is either 100 or 101? (4 points)

3. Let $ABCD$ be a rhombus. Suppose $APQC$ is a parallelogram such that the point B is inside $APQC$ and the side AP is equal to the side of the rhombus. Prove that B is the point where altitudes of triangle DPQ intersect, i.e. B is the orthocentre of triangle DPQ. (4 points)

4. Let n be an integer such that the equation $x^2 + y^2 + z^2 - xy - yz - zx = n$ has a solution in integers x, y, z. Prove that the equation $x^2 + y^2 - xy = n$ also has a solution in integers x, y. (5 points)

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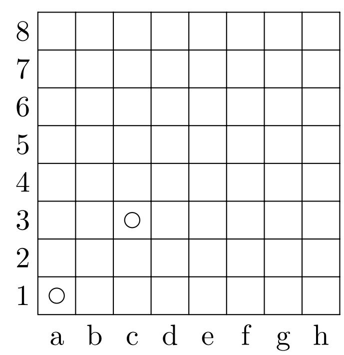
5. There are two identical draughts in the squares $a1$ and $c3$ of an 8×8 chessboard. Petya and Vasya make moves in turn under the following rules:

> Petya makes the first move.
> Each player can choose any draught and move it horizontally to the right or vertically upwards any number of squares.

The aim of each player is to place a draught in the square $h8$. Which player can always win for sure no matter how his opponent plays? (There may be only one draught in a square and draughts cannot jump over each other.)



(5 points)